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The Distribution Characteristics of ILS's Track Deviation Research

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***Abstract*—the distribution characteristics of the track deviation of the precision approach segment are studied. The Monte Carlo simulation is used to obtain the track deviation data when constructing the collision risk model [1]. Since it does not obey the Gaussian distribution, the GMM is established and the maximum expectation is used to obtain a probability density function. It is proved that 2-class GMM can describe the vertical deviation distribution and 3-class GMM can describe the horizontal deviation distribution. Comparing with the data in [1], the probability distribution constructing by this paper is proved to be accurate and effective. It also demonstrates the safety assessed by OAS for ultra-long runways.**

Keywords—Obstacle risk Assessment, Obstacle collision, Gaussian mixture model

I. INTRODUCTION

The probability analysis of collision risk between aircraft and surrounding obstacles is essentially to analyze that obstacles are located within the confidence interval of aircraft track distribution. The CRM model proposed by ICAO [1] is also based on the experimental statistics of aircraft distribution to obtain a method of evaluating the probability of collision with obstacles in the final stage of precision approach, and based on this, a simplified evaluation model, OAS [2], is established. One of the limits of OAS is that when the LLZ-THR exceeds 4500 m, the model uses an evaluation result of 4500m instead of an assessment greater than 4500m. Whether the security standards are met requires a restore of the probability density function, which is not currently provided by the ICAO.

Statistical methods are commonly used to deal with the distribution of flight tracks, which is the most commonly used means of analysis. However, the precision approach flight based on the instrument landing system (ILS) is different from that of other flight stages, with both horizontal deviation requirements and vertical deviation requirements. At the same time, the difference of meteorological conditions and the fluctuation of navigation signal have all the effects on flight's tracking, and the simple application of normal distribution model cannot guarantee the safety assessment.

In this paper, the simulation sample of ILS track deviation was obtained by Monte Carlo method to establish the Gaussian mixture model (GMM), and the parameter estimation was made by the Expectation-Maximization algorithm (EM), in order to obtain the mean and standard deviation between the proportion and distribution of the various Gauss distribution shape. Use the chi-square to verify the accuracy of the probability density function. The probability density function model obtained in this paper is used to assessment the QAR data samples obtained by ILS approach in Golmud airport of Qinghai province, China. The results prove that the use of OAS assessments at Golmud airport meets ICAO safety requirements.

II. CONSTRUCTION TRACK DEVIATION MODEL BASED ON MONTE CARLO METHOD

The ability of the aircraft to maintain a predetermined track is related to the beam transmitted by the ILS system, the onboard reception equipment and the control of the aircraft. Therefore, [1] uses the five variables of beam center error, sensitivity, receiver center error, receiver sensitivity and bending of the intersection of heading track and glide track, and one variable related to driving ability to construct the flight deviation model. The expression of each parameter is as follows:

- Beam cantering error (Φ_0):
- Beam sensitivity(K_1):
- Receiver centering error (I_0):
- Receiver sensitivity (K_2):
- Beam bends (BB):
- Piloting performance (I):

Considering the beam bending quantity BB, the indicated deviation of the airborne receiver is shown in formula 1:

$$I=I_0+K_1K_2(\Phi-\Phi_0)-BB \tag{1}$$

Φ parameter refers to the actual flight path deviation, and the detected deviation is $K_1(\Phi-\Phi_0)$. The deviation of the receiver is $K_2K_1(\Phi-\Phi_0)$, and the deviation detected by the actual receiver is $I_0+K_2K_1(\Phi-\Phi_0)$.Thus, the expression of the flight path deviation obtained by solving Φ is shown in formula (2) :

$$\Phi=(I-I_0+BB)/K_1K_2 \quad +\Phi_0 \tag{2}$$

The values of K1 and K2 are specified in ICAO annex 10. The nominal values of I_0, Φ_0 and BB are 0. However, its value varies in different locations. For distribution and parameters of the above variables, please refer to Attachment to Part II in [2]. However, only the available maximum nominal value of receiver sensitivity is given in Table [2], and its standard value can be determined by the median value of Table 11-3-6 in [1].

A. Monte Carlo Simulation

Based on the distribution law of variables in [1, 2], Monte Carlo method is adopted to generate random Numbers of each variable, and then the simulation results of track deviation are obtained by using formula 3.

$$P(x|\theta) = \prod_{i=1}^{N} P\left(x^{(i)}|\theta\right) = L(\theta|x) \qquad (3)$$

$L(\theta\ |X)$ is the likelihood function of the sample X, which can be seen as a constant and the parameter θ as a variable function.

The algorithm is as follows:

Step 0: Substitute the maximum nominal value of receiver sensitivity;

Step 1: Generate random Numbers of each variable;

Step 2: Put into the model to get simulation results, and cycle 2000 times to get 2000 track deviations;

Step 3: Calculate the standard deviation of track deviation to determine whether it is similar to the data in literature [1]. If it is consistent, the simulation ends; otherwise, the nominal value of receiver sensitivity is reduced.

Fig. 1 shows the track deviation simulation results at 7800m from the end of the runway.

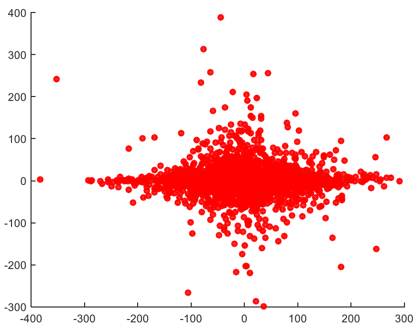
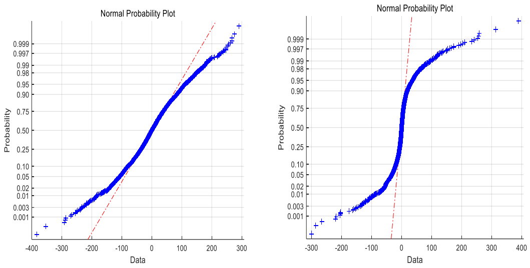


Fig. 1 Course deviation distribution

According to the simulation results shown in Fig. 1, the course track distribution is wide, while the glide track distribution is narrow, and the overall distribution is similar to the CRM equal-probability elliptic curve. The narrow vertical distribution is because the AP-based flight mode is more precise to control the altitude deviation of the aircraft, and the pilot is more sensitive to the altitude deviation, and makes timely changes to the deviation. Based on this, the obtained simulation data is reliable.

B. Probability distribution model analysis

The acquired data are analyzed for probability distribution. The normal distribution probability of the data is shown in Fig. 2. The blue point is the track deviation and the red line is the normal distribution. Fig. 2a is the distribution probability of horizontal track deviation, and Fig. 2b is the distribution probability of vertical track deviation. It can be seen from the figure that both the horizontal distribution and the vertical distribution are significantly different from the normal distribution.



(a) (b)

Fig. 2 probability diagram of horizontal (vertical) track deviation distribution

Subsequently, chi-square test was used to verify whether it met the normal distribution, and the results did not obey the confidence of 99.5% Gaussian distribution. Compared with the horizontal distribution, the vertical distribution is significantly different, indicating that the pilot's ability to maintain altitude is more obvious.

III. ESTABLISHMENT OF GMM

The Gaussian Probability Density Function with a single mean value and standard deviation cannot meet the requirements of Function description, so it is considered to use GMM to solve this problem. In other words, it is considered to use multiple Gaussian Probability Density functions to fit the track deviation [3-6].However, it is difficult for GMM model to use moment estimation and maximum likelihood estimation method to solve the problem of individual Gaussian distribution parameters, while EM algorithm is widely used in the field of machine learning to solve this problem. Therefore, this paper attempts to use EM algorithm to estimate unknown parameters in GMM.

To apply the GMM, we need to specify which types of Gaussian the sample belongs to. If the classification is less, the fitting degree is still not high and the probability distribution cannot be correctly reflected. If there are too many classifications, the expression of probability density function will be too complex, which is not conducive to the subsequent calculation of collision risk probability and the portability and applicability will also be greatly affected. Therefore, Gaussian distribution should be carefully selected to meet the accuracy and conciseness of data description by probability density function.

A. Two class of GMM

Since the horizontal and vertical deviation distributions of the track are independent of each other, the data distributions in the horizontal and vertical directions will be taken into account in the establishment of the model. Firstly, the two discriminations are constructed into a 2-class GMM, as shown in formula 4 and 5.

$$f_l(x_l) = \frac{\phi_{l1}}{\sqrt{2\pi}\sigma_{l1}} \exp\left(-\frac{(x_l-\mu_{l1})^2}{2\sigma_{l1}^2}\right) + \frac{(1-\phi_{l1})}{\sqrt{2\pi}\sigma_{l2}} \exp\left(-\frac{(x_l-\mu_{l2})^2}{2\sigma_{l2}^2}\right) \qquad (4)$$

$$f_v(x_v)=\frac{\phi_{v1}}{\sqrt{2\pi}\sigma_{v1}}\exp\left(-\frac{(x_v-\mu_{v1})^2}{2\sigma_{v1}^2}\right)+\frac{(1-\phi_{v1})}{\sqrt{2\pi}\sigma_{v2}}\exp\left(-\frac{(x_v-\mu_{v2})^2}{2\sigma_{v2}^2}\right) \quad (5)$$

In the formula, $f_l(x_l)$ and $f_v(x_v)$ represents horizontal deviation x_l and vertical deviation x_v probability density function. Φ_{l1} and Φ_{v1} represents probability that represents horizontal or vertical sample data belonging to the first class Gaussian distribution. μ_{l1},μ_{l2},μ_{v1} and μ_{v2} represents each corresponding expectation. σ_{l1},σ_{l2},σ_{v1} and σ_{v2} represents each corresponding standard deviation.

The EM algorithm mentioned above is applied to estimate the above parameters. The update of the lower bound of the logarithmic likelihood function with the number of iterations is shown in figure 3.

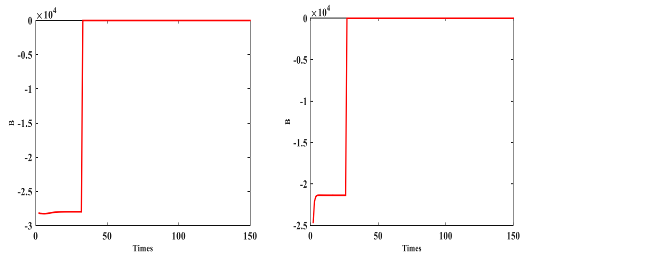


Fig. 3 The relation between the 2-class GMM iterations and the lower bound

The estimated values of each parameter are shown in table I. The EM algorithm converges horizontally and vertically through 36 and 24 steps respectively. Values of each parameter after algorithm convergence are shown in Table ii-att-1/2 in [2].

TABLE I 2-CLASS GMM (UNIT: M)

Parameters	Value	Parameters	Value
Φ_{l1}	0.4992	μ_{v2}	1.22
Φ_{v1}	0.6407	σ_{l1}	38.7
μ_{l1}	-1.43	σ_{l2}	86.5
μ_{l2}	2.90	σ_{v1}	28.0
μ_{v1}	-0.11	σ_{v2}	46.86

The difference of the two types of standard deviations in the vertical distribution was significant, indicating that the difference of the vertical track retention ability was significant. And most of the Φ_{v1}= 0.6407 vertical track maintenance ability is good.

Then the fitting accuracy of two kinds of GMM is judged, as shown in Fig. 4, which is the probability distribution histogram and probability density function of track deviation, and the left side is the horizontal direction and the right side is the vertical direction.

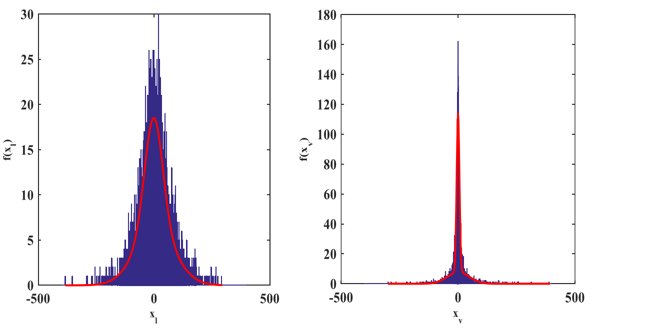


Fig. 4 probability density function and histogram of two kinds of Gaussian models

It can be observed from Fig. 4 that the probability density function of the track vertical deviation distribution fits well and can pass the chi-square test with confidence of 99.5%. However, the fitting degree of probability density function of the left track horizontal deviation distribution is not perfect, and it fails to pass the chi-square test.

B. Three-class GMM

Aiming at the problem that the 2-class GMM cannot well fit the horizontal track deviation, this paper tries to increase and decrease the categories of Gaussian model. Firstly, 3-class GMM are tried to fit the horizontal track deviation. The estimated values of each parameter are shown in table II.

TABLE II 3-CLASS GMM (UNIT: M)

Parameters	Value	Parameters	Value
Φ_{l1}	0.3323	-	-
Φ_{v1}	0.3349	-	-
μ_{l1}	-3.34	σ_{l1}	33.05
μ_{l2}	5.19	σ_{l2}	60.80
μ_{l3}	0.33	σ_{l3}	93.17

μ_{l3} is the expectation of the third type, and σ_{l3} is the standard deviation of the third type. Thus, probability density functions of three Gaussian distributions and histogram of horizontal track deviation data distribution are drawn, as shown in Fig. 5:

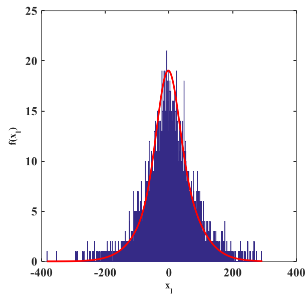


Fig. 5 probability density function and histogram of 3-class of GMM

It can be seen from Fig. 5 that, compared with the 2-class GMM in Fig. 4, the fitting accuracy of the 3-class GMM is significantly increased, and the hypothesis test of the distribution probability density function shows that it can pass the chi-square test with a confidence of 99.95.

Therefore, it is proved that 3-class GMM can be used to describe the probability density function of horizontal track deviation, and 2-class GMM can be used to describe the probability density function of vertical track, which is significantly higher than the traditional Gaussian distribution fitting accuracy.

The fitting degree of any kind of Gaussian distribution in the track deviation of fitting is mainly at the peak and tail is seriously low. However, the data of this simulation model are obtained based on the statistics of the 1980s, and the track deviation distribution is obviously large. However, due to the improvement of aircraft track performance, the track deviation distribution has been changed. In order to facilitate future probability calculation, it is necessary to prove that the GMM is still applicable at present.

IV. Model Validation

According to the above studies, compared with the single Gaussian distribution, the probability distribution obtained by GMM and EM can better fit the horizontal or vertical track deviation. However, in order to make the calculation results of probability of subsequent collision risk credible enough, it is necessary to verify the matching degree with the data in CRM. By analogy with the format of the probability table in [1], the standard deviation of integer multiples is selected for verification, where the standard deviation should be the weighted value of the standard deviation, as shown in formula 6, where is the multiple of the standard deviation, and is the number of categories divided by GMM. The values of both are cumulative probabilities on the positive side of the nominal track, with the horizontal direction being right-biased and the vertical direction being right-biased. Thus, the statistics are shown in table III.

$$n\sigma^{i}(x) = n\sum_{i=1}^{k} \varphi^{i} \times \sigma^{i}(x) \qquad (6)$$

TABLE III Probability value comparison.

Standard difference multiple n	Horizontal probability	CRM horizontal probability	Vertical probability	CRM Vertical probability
0	4.985×10^{-1}	5.000×10^{-1}	5.027×10^{-1}	5.000×10^{-1}
1	1.301×10^{-1}	1.371×10^{-1}	1.533×10^{-1}	1.422×10^{-1}
2	2.780×10^{-2}	2.642×10^{-2}	3.010×10^{-2}	2.660×10^{-2}
3	4.500×10^{-3}	4.621×10^{-3}	4.414×10^{-3}	4.414×10^{-3}
4	5.004×10^{-4}	5.526×10^{-4}	5.100×10^{-4}	5.232×10^{-4}
5	4.500×10^{-5}	3.257×10^{-5}	4.114×10^{-5}	3.543×10^{-5}
6	1.4059×10^{-6}	1.017×10^{-6}	1.718×10^{-6}	1.467×10^{-6}
7	3.383×10^{-8}	2.862×10^{-8}	4.230×10^{-8}	4.532×10^{-8}

It can be seen from the above table that the cumulative probability value obtained by GMM is basically similar to that recorded by [1], and the collision risk probability obtained by this method is accurate and effective.

In order to further verify the reliability of the model. This paper selects a typical plateau airport in China for model verification and evaluation. Qinghai Golmud airport runway 27 LLZ-THR is 4800 meters. In this paper, 400 ILS approaches of different aircraft types in a year in Golmud airport are selected as research objects. Data from the airborne QAR. Fig. 6 shows the scatter diagram of track deviation at 7800m.

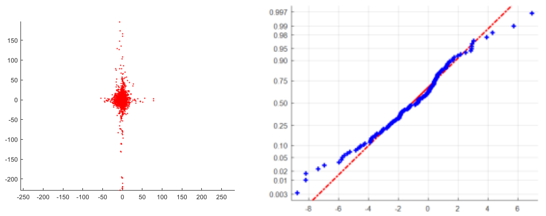


Fig. 6 Golmud airport ILS approach track deviation distribution

Fig. 6 shows that, compared with the simulation data and the Golmud airport ILS operational data, the distribution shape of the two is similar. From the morphological distribution of the deviation, the actual approach deviation value is small, and the vertical deviation distribution range is larger than the horizontal deviation distribution range. Although the deviation of track is different from the simulation data, it still satisfies the 2-class GMM distribution in the vertical direction and 3-class in the horizontal direction. The reason is that the aircraft mainly apply the autopilot function when making precise approach, so the horizontal track is maintained well. Golmud airport is a plateau airport the true airspeed of the aircraft's final approach is higher than that of the sea level, and the deviation caused by the maintenance of vertical track is larger. This indicates that the ability of aircraft to maintain the scheduled flight path has been greatly improved compared with the introduction of CRM.

Based on the flight path deviation analysis of the approach of Golmud airport, it can be seen that it is safe to use the OAS of 4500 meters to evaluate obstacles for the runway exceeding 4500 meters of LLZ-THR.

V. Conclusions

There are more and more plateau airports in China, and the runway length is getting longer and longer according to the requirements of aircraft performance. When the LLZ-THR exceeds 4500 meters, the OAS evaluation model provided by ICAO can only replace the one over 4500 meters with 4500 meters. In this paper, the probability distribution density function of track deviation of CRM model is obtained by Monte Carlo simulation. By establishing GMM model and using EM algorithm to estimate the parameters, it is proved that 2-class GMM can describe the vertical deviation distribution of track and 3-class GMM can describe the horizontal deviation distribution of track. Through comparison with the data in [1], the method in this paper is proved to be accurate and effective. Through the data evaluation of ILS approach of Golmud airport in Qinghai province, China, it proves that the ILS operation at Golmud airport is safe and reliable.

Acknowledgment

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